

### **General Certificate of Education**

## **Mathematics 6360**

MM2B Mechanics 2B

# **Mark Scheme**

2008 examination - June series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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#### Key to mark scheme and abbreviations used in marking

M	mark is for method			
m or dM	mark is dependent on one or more M marks and is for method			
A	mark is dependent on M or m marks and is for accuracy			
В	mark is independent of M or m marks and is for method and accuracy			
E	mark is for explanation			
or ft or F	follow through from previous			
	incorrect result	MC	mis-copy	
CAO	correct answer only	MR	mis-read	
CSO	correct solution only	RA	required accuracy	
AWFW	anything which falls within	FW	further work	
AWRT	anything which rounds to	ISW	ignore subsequent work	
ACF	any correct form	FIW	from incorrect work	
AG	answer given	BOD	given benefit of doubt	
SC	special case	WR	work replaced by candidate	
OE	or equivalent	FB	formulae book	
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme	
–x EE	deduct x marks for each error	G	graph	
NMS	no method shown	c	candidate	
PI	possibly implied	sf	significant figure(s)	
SCA	substantially correct approach	dp	decimal place(s)	

#### No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

### MM2B

Q	Solution	Marks	Total	Comments
1(a)	$a = \frac{\mathrm{d}v}{\mathrm{d}t} = 12t + 4$	M1 A1	2	
	$\mathrm{d}t$			
(b)	Using $F = ma$ ,			
(6)	Force = $3 \times (12t + 4)$	M1		
	When $t = 4$ , force = 3 (12 × 4 + 4)			
	Force = $156 \text{ N}$	A1	2	
(c)	$r = 2t^3 + 2t^2 - 7t + c$	M1 A1		
	When $t = 0$ , $r = 5$ , $\therefore c = 5$	M1		
	$\therefore r = 2t^3 + 2t^2 - 7t + 5$	A1	4	SC3 if no '+c' seen
	Total	711	8	Ses if no it seen
2(a)	$\uparrow T_A$ $\uparrow T_B$	B1	1	
	$\frac{1}{A}$ $\sqrt{40g}$ $\frac{1}{B}$			
	<b>▼</b> 40g			
(b)	Taking moments about A			
(0)	$2.1 \times 40g = T_B \times 4$	M1 B1		B1 for 2.1
	$T_B = 21g$	A1	3	B1 101 2.1
	•			
(c)	Resolve vertically $T_A + T_B = 40g$	M1	2	
	$T_A = 19g  \text{or}  186 \text{ N}$	A1	2	
(d)	Gravitational force acts through mid point	E1	1	
	of the rod			
3	75 × 1 + 12 × 4 + 4 × 5		7	
3	$\overline{X} = \frac{25 \times 1 + 12 \times 4 + 4 \times 5}{1 + 4 + 5}$	M1		Two terms on top correct (+third) and
	1+4+3			denominator correct
	$=\frac{93}{10}$ or 9.3	A 1		
	10	A1		
	$\overline{Y} = \frac{10 \times 1 + 7 \times 4 + 18 \times 5}{10 \times 10^{-1}}$	) / 1		
	$Y = \frac{10}{10}$	M1		
	$=\frac{128}{10}$ or 12.8	A1	4	$SC3$ for interchanged $\overline{V}$ and $\overline{V}$
	10	Al	4	SC3 for interchanged $\bar{X}$ and $\bar{Y}$
	∴ Centre of mass is at (9.3, 12.8)			
	Total		4	

MM2B (cont)

Q	t) Solution	Marks	Total	Comments
4(a)	Using power = force $\times$ velocity			
	$Power = (40 \times 50) \times 50$	M1		
	$\therefore = 100,000 \text{ watts}$	A1	2	
(b)	When speed is 25,			
(~)	•			
	max force exerted is $\frac{100000}{25}$			
	= 4000N	B1		
	: Accelerating force is 3000N			
	Using $F = ma$ 3000 = 1500 a	M1		Need 3 terms eg '4000' $\pm 1000 = ma$
	3000 1300 <b>u</b>	1411		or $2000 \pm 1000 = ma$
				M0 for $1000 = ma$
	$a = 2 \text{ ms}^{-2}$	A1	3	
(c)	When van is at maximum speed	D1		
	force against gravity is <i>mg</i> sin 6 (parallel to slope)	B1		
	Force against gravity and resistance is			
	$mg \sin 6 + 40 v$	M1		
	= 1536.6 + 40 v	A1		
	Speed is maximum			
	when $1536.6 + 40v = \frac{100000}{v}$	M1		For 3 terms; $\frac{100000}{3}$ and 1 other term
	ν			correct
	$40 v^2 + 1536.6 v - 100 000 = 0$	A1		CAO
	Speed is 34.4 ms <sup>-1</sup>	A1	6	
1	Speed is 54.4 ms	A1	U	
	Total	Al	11	
5(a)	Total	Al		
	$\mathbf{v} = \frac{\mathbf{dr}}{\mathbf{d}t}$	Al		
	$\mathbf{v} = \frac{\mathbf{dr}}{\mathbf{d}t}$	M1 A1		No <b>i</b> , <b>j</b> : no marks
	$\mathbf{v} = \frac{\mathbf{dr}}{\mathbf{d}t}$		11	No i, j: no marks
	$\mathbf{v} = \frac{\mathbf{dr}}{\mathbf{d}t}$		11	No i, j: no marks
	Total $\mathbf{v} = \frac{d\mathbf{r}}{dt}$ $\mathbf{v} = -2\sin\frac{1}{4}t\mathbf{i} - 2\cos\frac{1}{4}t\mathbf{j}$ Speed is $\{(-2\sin\frac{1}{4}t)^2 + (-2\cos\frac{1}{4}t)^2\}^{\frac{1}{2}}$	M1 A1	11	
	Total $\mathbf{v} = \frac{d\mathbf{r}}{dt}$ $\mathbf{v} = -2\sin\frac{1}{4}t\mathbf{i} - 2\cos\frac{1}{4}t\mathbf{j}$ Speed is $\{(-2\sin\frac{1}{4}t)^2 + (-2\cos\frac{1}{4}t)^2\}^{\frac{1}{2}}$	M1 A1	11	No <b>i</b> , <b>j</b> : no marks  clear use of $\sin^2 \theta + \cos^2 \theta = 1$
	Total $\mathbf{v} = \frac{d\mathbf{r}}{dt}$ $\mathbf{v} = -2\sin\frac{1}{4}t\mathbf{i} - 2\cos\frac{1}{4}t\mathbf{j}$ Speed is $\{(-2\sin\frac{1}{4}t)^2 + (-2\cos\frac{1}{4}t)^2\}^{\frac{1}{2}}$ $= 2\left(\sin^2\frac{1}{4}t + \cos^2\frac{1}{4}t\right)^{\frac{1}{2}}$	M1 A1 M1 m1	2	clear use of $\sin^2 \theta + \cos^2 \theta = 1$
	Total $\mathbf{v} = \frac{d\mathbf{r}}{dt}$ $\mathbf{v} = -2\sin\frac{1}{4}t\mathbf{i} - 2\cos\frac{1}{4}t\mathbf{j}$ Speed is $\{(-2\sin\frac{1}{4}t)^2 + (-2\cos\frac{1}{4}t)^2\}^{\frac{1}{2}}$	M1 A1	11	
(b)	Total $\mathbf{v} = \frac{d\mathbf{r}}{dt}$ $\mathbf{v} = -2\sin\frac{1}{4}t\mathbf{i} - 2\cos\frac{1}{4}t\mathbf{j}$ Speed is $\{(-2\sin\frac{1}{4}t)^2 + (-2\cos\frac{1}{4}t)^2\}^{\frac{1}{2}}$ $= 2\left(\sin^2\frac{1}{4}t + \cos^2\frac{1}{4}t\right)^{\frac{1}{2}}$ $= 2 \text{ which is a constant}$ Magnitude of $\mathbf{r}$ is	M1 A1 M1 m1 A1	2	clear use of $\sin^2 \theta + \cos^2 \theta = 1$ Use of 2 values SC1
(b)	Total $\mathbf{v} = \frac{d\mathbf{r}}{dt}$ $\mathbf{v} = -2\sin\frac{1}{4}t\mathbf{i} - 2\cos\frac{1}{4}t\mathbf{j}$ Speed is $\{(-2\sin\frac{1}{4}t)^2 + (-2\cos\frac{1}{4}t)^2\}^{\frac{1}{2}}$ $= 2\left(\sin^2\frac{1}{4}t + \cos^2\frac{1}{4}t\right)^{\frac{1}{2}}$ $= 2 \text{ which is a constant}$	M1 A1 M1 m1	2	clear use of $\sin^2 \theta + \cos^2 \theta = 1$
(b)	Total $\mathbf{v} = \frac{d\mathbf{r}}{dt}$ $\mathbf{v} = -2\sin\frac{1}{4}t\mathbf{i} - 2\cos\frac{1}{4}t\mathbf{j}$ Speed is $\{(-2\sin\frac{1}{4}t)^2 + (-2\cos\frac{1}{4}t)^2\}^{\frac{1}{2}}$ $= 2\left(\sin^2\frac{1}{4}t + \cos^2\frac{1}{4}t\right)^{\frac{1}{2}}$ $= 2 \text{ which is a constant}$ Magnitude of $\mathbf{r}$ is	M1 A1 M1 m1 A1	2	clear use of $\sin^2 \theta + \cos^2 \theta = 1$ Use of 2 values SC1
(b)	Total $\mathbf{v} = \frac{d\mathbf{r}}{dt}$ $\mathbf{v} = -2\sin\frac{1}{4}t\mathbf{i} - 2\cos\frac{1}{4}t\mathbf{j}$ Speed is $\{(-2\sin\frac{1}{4}t)^2 + (-2\cos\frac{1}{4}t)^2\}^{\frac{1}{2}}$ $= 2\left(\sin^2\frac{1}{4}t + \cos^2\frac{1}{4}t\right)^{\frac{1}{2}}$ $= 2 \text{ which is a constant}$ Magnitude of $\mathbf{r}$ is $\{(8\cos\frac{1}{4}t)^2 + (8\sin\frac{1}{4}t)^2\}^{\frac{1}{2}}$	M1 A1 M1 A1 M1	2 3	clear use of $\sin^2 \theta + \cos^2 \theta = 1$ Use of 2 values SC1
(b)	Total $\mathbf{v} = \frac{d\mathbf{r}}{dt}$ $\mathbf{v} = -2\sin\frac{1}{4}t\mathbf{i} - 2\cos\frac{1}{4}t\mathbf{j}$ Speed is $\{(-2\sin\frac{1}{4}t)^2 + (-2\cos\frac{1}{4}t)^2\}^{\frac{1}{2}}$ $= 2\left(\sin^2\frac{1}{4}t + \cos^2\frac{1}{4}t\right)^{\frac{1}{2}}$ $= 2 \text{ which is a constant Magnitude of } \mathbf{r} \text{ is}$ $\{(8\cos\frac{1}{4}t)^2 + (8\sin\frac{1}{4}t)^2\}^{\frac{1}{2}}$ $= 8 \text{ which is a constant}$ $\therefore \text{ Particle is moving in a circle}$	M1 A1 M1 A1 A1	2 3	clear use of $\sin^2 \theta + \cos^2 \theta = 1$ Use of 2 values SC1 $\mathbf{a} = -k\mathbf{r} \Rightarrow \text{circle}  \text{SC2}$
(b) (c)	Total $\mathbf{v} = \frac{d\mathbf{r}}{dt}$ $\mathbf{v} = -2\sin\frac{1}{4}t\mathbf{i} - 2\cos\frac{1}{4}t\mathbf{j}$ Speed is $\{(-2\sin\frac{1}{4}t)^2 + (-2\cos\frac{1}{4}t)^2\}^{\frac{1}{2}}$ $= 2\left(\sin^2\frac{1}{4}t + \cos^2\frac{1}{4}t\right)^{\frac{1}{2}}$ $= 2 \text{ which is a constant Magnitude of } \mathbf{r} \text{ is }$ $\{(8\cos\frac{1}{4}t)^2 + (8\sin\frac{1}{4}t)^2\}^{\frac{1}{2}}$ $= 8 \text{ which is a constant }$ $\therefore \text{ Particle is moving in a circle}$ Using $v = a\omega$	M1 A1 M1 A1 M1 A1	2 3	clear use of $\sin^2 \theta + \cos^2 \theta = 1$ Use of 2 values SC1
(b) (c) (d)	Total $\mathbf{v} = \frac{d\mathbf{r}}{dt}$ $\mathbf{v} = -2\sin\frac{1}{4}t\mathbf{i} - 2\cos\frac{1}{4}t\mathbf{j}$ Speed is $\{(-2\sin\frac{1}{4}t)^2 + (-2\cos\frac{1}{4}t)^2\}^{\frac{1}{2}}$ $= 2\left(\sin^2\frac{1}{4}t + \cos^2\frac{1}{4}t\right)^{\frac{1}{2}}$ $= 2 \text{ which is a constant}$ Magnitude of $\mathbf{r}$ is $\{(8\cos\frac{1}{4}t)^2 + (8\sin\frac{1}{4}t)^2\}^{\frac{1}{2}}$ $= 8 \text{ which is a constant}$ $\therefore \text{ Particle is moving in a circle}$ Using $v = a\omega$ Angular speed is 0.25	M1 A1 M1 A1 A1	2 3	clear use of $\sin^2 \theta + \cos^2 \theta = 1$ Use of 2 values SC1 $\mathbf{a} = -k\mathbf{r} \Rightarrow \text{circle}  \text{SC2}$
(b) (c)	Total $\mathbf{v} = \frac{d\mathbf{r}}{dt}$ $\mathbf{v} = -2\sin\frac{1}{4}t\mathbf{i} - 2\cos\frac{1}{4}t\mathbf{j}$ Speed is $\{(-2\sin\frac{1}{4}t)^2 + (-2\cos\frac{1}{4}t)^2\}^{\frac{1}{2}}$ $= 2\left(\sin^2\frac{1}{4}t + \cos^2\frac{1}{4}t\right)^{\frac{1}{2}}$ $= 2 \text{ which is a constant}$ Magnitude of $\mathbf{r}$ is $\{(8\cos\frac{1}{4}t)^2 + (8\sin\frac{1}{4}t)^2\}^{\frac{1}{2}}$ $= 8 \text{ which is a constant}$ $\therefore \text{ Particle is moving in a circle}$ Using $v = a\omega$ Angular speed is 0.25	M1 A1 M1 A1 M1 A1	2 3	clear use of $\sin^2 \theta + \cos^2 \theta = 1$ Use of 2 values SC1 $\mathbf{a} = -k\mathbf{r} \Rightarrow \text{circle}  \text{SC2}$
(b) (c) (d) (e)	Total $\mathbf{v} = \frac{d\mathbf{r}}{dt}$ $\mathbf{v} = -2\sin\frac{1}{4}t\mathbf{i} - 2\cos\frac{1}{4}t\mathbf{j}$ Speed is $\{(-2\sin\frac{1}{4}t)^2 + (-2\cos\frac{1}{4}t)^2\}^{\frac{1}{2}}$ $= 2\left(\sin^2\frac{1}{4}t + \cos^2\frac{1}{4}t\right)^{\frac{1}{2}}$ $= 2 \text{ which is a constant Magnitude of } \mathbf{r} \text{ is }$ $\{(8\cos\frac{1}{4}t)^2 + (8\sin\frac{1}{4}t)^2\}^{\frac{1}{2}}$ $= 8 \text{ which is a constant }$ $\therefore \text{ Particle is moving in a circle }$ Using $v = a\omega$ Angular speed is 0.25 $\mathbf{a} = -\frac{1}{2}\cos\frac{1}{4}t\mathbf{i} + \frac{1}{2}\sin\frac{1}{4}t\mathbf{j}$	M1 A1 M1 A1 M1 A1 M1 A1 M1 A1	2 3 2 2 2 2	clear use of $\sin^2 \theta + \cos^2 \theta = 1$ Use of 2 values SC1 $\mathbf{a} = -k\mathbf{r} \Rightarrow \text{circle}  \text{SC2}$
(b) (c) (d)	Total $\mathbf{v} = \frac{d\mathbf{r}}{dt}$ $\mathbf{v} = -2\sin\frac{1}{4}t\mathbf{i} - 2\cos\frac{1}{4}t\mathbf{j}$ Speed is $\{(-2\sin\frac{1}{4}t)^2 + (-2\cos\frac{1}{4}t)^2\}^{\frac{1}{2}}$ $= 2\left(\sin^2\frac{1}{4}t + \cos^2\frac{1}{4}t\right)^{\frac{1}{2}}$ $= 2 \text{ which is a constant}$ Magnitude of $\mathbf{r}$ is $\{(8\cos\frac{1}{4}t)^2 + (8\sin\frac{1}{4}t)^2\}^{\frac{1}{2}}$ $= 8 \text{ which is a constant}$ $\therefore \text{ Particle is moving in a circle}$ Using $v = a\omega$ Angular speed is 0.25	M1 A1 M1 A1 M1 A1 A1	2 3 2	clear use of $\sin^2 \theta + \cos^2 \theta = 1$ Use of 2 values SC1 $\mathbf{a} = -k\mathbf{r} \Rightarrow \text{circle}  \text{SC2}$

MM2B (cont)

Q Q	Solution	Marks	Total	Comments
6(a)	Using $F = ma$			
	$-0.05mv = m \frac{dv}{dx}$			
	$\mathrm{d}t$	B1	1	N. 14
	$-0.05mv = m \frac{dv}{dt}$ $\therefore \frac{dv}{dt} = -0.05v$ $\int \frac{dv}{v} = -\int 0.05 dt$	Б1	1	Need to see <i>m</i> terms
(b)	$\int dv = \int 0.05 dt$	B1		
	$\int \frac{1}{v} = \int 0.03  dt$			
	$ \ln v = -0.05t + c $	M1		Need first 2 terms
	$v = Ce^{-0.05t}$			
	When $t = 0$ , $v = 20$ , ∴ $C = 20$	M1		٦
	$v = 20e^{-0.05t}$		4	fully correct solutions
	$v = 20e^{-4\pi i t}$	A1	4	J
(c)	When $v = 10$ , $10 = 20e^{-0.05t}$	M1		
	$e^{0.05t} = 2$	A1		
	$\therefore t = \frac{1}{0.05} \ln 2$			
	= 13.9	A1	3	Accept 20 ln 2
7(a)	At top, for complete revolutions:		8	
7(a)				
	$\frac{mv^2}{a} = mg$ where v is speed at top	M1		
	$\therefore v^2 = ag$	A1		
	Conservation of energy from <i>B</i> to top :			
	$\frac{1}{2}mv^2 + mg2a = \frac{1}{2}mu^2$	M1		3 terms, 2 KE and PE
	_	A1		
	$u^2 = 4ag + v^2$			
	= 5ag			
	$u=\sqrt{5ag}$	A1	5	AG
(b)	At C, speed of particle is $\sqrt{3ag}$	B1		
	Resolving horizontally at C:	3.55		N 10
	$T = \frac{mv^2}{a}$	M1		Needs 2 correct terms
	a 2 a a			
	$T = m \frac{3ag}{a}$			
	T = 3mg	A1	3	
(c)	No air resistance	B1	1	
	Bead is a particle		0	
	Total		9	

MM2B (cont)

Q Q	Solution	Marks	Total	Comments
8(a)	$\frac{e}{\zeta \lambda x}$			
	Work done = $\int_{-\infty}^{e} \frac{\lambda x}{l} dx$	M1		
	0			
	$=\left[\lambda x^{2}\right]^{e}$	A1		Needs limit of 0
	$= \left[\frac{\lambda x^2}{2l}\right]_0^e$	AI		Needs little of 0
	$= \frac{\lambda e^2}{2l}$	A1	3	AG
	Or			
	Area under a straight line =			
	average force $\times$ distance $=\frac{\lambda e^2}{2I}$			
(F)(;)	_,			
(b)(i)	Using $T = \frac{\lambda x}{l}$			
	$5g = \frac{150 \times x}{0.6}$	M1		
	0.0		2	
	Extension is 0.196 m	A1	2	
(ii)	$\lambda x^2$			
	$EPE = \frac{\lambda x^2}{2l}$			
	$= \frac{150 \times (0.3)^2}{2 \times 0.6}$	M1		
	= 11.25 J	A1	2	
(iii)	When $x$ above $P$ ,			
()				$150 \times ( x)^2$
	EPE = $\frac{150 \times (0.3 - x)^2}{2 \times 0.6}$	M1		for $\frac{150 \times ( x)^2}{2 \times 0.6}$
	PE[relative to $P$ ] = $(-)5 \times g \times x$	A1 M1		for $5 \times g \times \text{distance}$
	TELEBRITYE TO $I = (-)3 \wedge g \wedge x$	1V1 1		101 5 ^ g ^ distance
	KE + EPE [at new point]			
	= EPE [at P] - gain in PE	M1		4 terms, all signs correct, 2 terms correct
	$\frac{1}{2}mv^2 + \frac{150 \times (0.3 - x)^2}{2 \times 0.6} =$			
	2 2 1 0.0			
	$\frac{150\times(0.3)^2}{2\times0.6}-5gx$	A1		
	=			
	$\frac{1}{2}mv^2 + \frac{150 \times (x^2 - 0.6x)}{2 \times 0.6} = -5gx$	m1		Equation involving terms in $v^2$ , $x^2$ and $x$
				only
	$\frac{1}{2}.5.v^2 + 125 x^2 - 75 x = -49x$ $v^2 = 10.4x - 50 x^2$			
	$v^2 = 10.4x - 50 x^2$	A1	7	
(:-)	Particle is at rest when $v = 0$			
(iv)	$10.4x - 50 x^2 = 0$	M1		
	x = 0 [not required]			
	Or $x = \frac{10.4}{50} = 0.208$ m above <i>P</i> .	A1	2	
	Total TOTAL		16 75	
	IUIAL	<u> </u>	13	<u> </u>